

$$\begin{aligned} \boxed{1} \quad (1) \quad (\text{与式}) &= (\sqrt{2^2 \cdot 3} + \sqrt{5})(\sqrt{4^2 \cdot 3} - \sqrt{2^2 \cdot 5}) \\ &= (2\sqrt{3} + \sqrt{5})(4\sqrt{3} - 2\sqrt{5}) \\ &= 2(2\sqrt{3} + \sqrt{5})(2\sqrt{3} - \sqrt{5}) \\ &= 2\{(2\sqrt{3})^2 - (\sqrt{5})^2\} \\ &= 2(12 - 5) = \text{ア}14 \end{aligned}$$

$$\begin{aligned} (2) \quad (\text{与式}) &= \{\sqrt{2} + (\sqrt{3} + \sqrt{6})\}\{\sqrt{2} - (\sqrt{3} + \sqrt{6})\} \\ &= (\sqrt{2})^2 - (\sqrt{3} + \sqrt{6})^2 \\ &= 2 - (3 + 2\sqrt{3}\sqrt{6} + 6) \\ &= \text{イ} - 7 - 6\sqrt{2} \end{aligned}$$

$$\begin{aligned} (3) \quad (\text{与式}) &= \frac{\sqrt{3}(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)} - \frac{\sqrt{2}(\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} - \frac{2(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} \\ &= \frac{\sqrt{3}(\sqrt{2} + 1)}{2 - 1} - \frac{\sqrt{2}(\sqrt{3} - \sqrt{2})}{3 - 2} - \frac{2(\sqrt{3} + 1)}{3 - 1} \\ &= \sqrt{6} + \sqrt{3} - (\sqrt{6} - 2) - (\sqrt{3} + 1) \\ &= \text{ウ}1 \end{aligned}$$

$$\begin{aligned} (4) \quad 2 - \sqrt{3} &= \sqrt{4} - \sqrt{3} > 0, \quad 3 - 2\sqrt{3} = \sqrt{9} - \sqrt{12} < 0 \text{ であるから} \\ \sqrt{(2 - \sqrt{3})^2} + \sqrt{(3 - 2\sqrt{3})^2} &= 2 - \sqrt{3} - (3 - 2\sqrt{3}) \\ &= \text{エ} - 1 + \sqrt{3} \end{aligned}$$

$$\begin{aligned} \boxed{2} \quad x + y &= \frac{1}{\sqrt{5} + \sqrt{3}} + \frac{1}{\sqrt{5} - \sqrt{3}} \\ &= \frac{(\sqrt{5} - \sqrt{3}) + (\sqrt{5} + \sqrt{3})}{(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})} = \frac{2\sqrt{5}}{5 - 3} \\ &= \text{ア} \sqrt{5} \end{aligned}$$

$$xy = \frac{1}{\sqrt{5} + \sqrt{3}} \cdot \frac{1}{\sqrt{5} - \sqrt{3}} = \frac{1}{5 - 3} = \text{イ} \frac{1}{2}$$

$$\text{よって} \quad x^2 + y^2 = (x + y)^2 - 2xy = (\sqrt{5})^2 - 2 \cdot \frac{1}{2} = \text{ウ}4$$

$$x^3y + xy^3 = xy(x^2 + y^2) = \frac{1}{2} \cdot 4 = \text{エ}2$$

$\boxed{3}$  数直線上の2点  $B(b)$ ,  $C(c)$  間の距離  $BC$  は

$$b \leq c \text{ のとき} \quad BC = c - b$$

$$b > c \text{ のとき} \quad BC = b - c = -(c - b)$$

これを絶対値の記号を用いて表すと  $BC = |c - b|$

$\boxed{\text{別解}}$  数直線上の2点  $B(b)$ ,  $C(c)$  間の距離は、それぞれの点を  $-c$  だけ平行移動した2点  $D(b - c)$ ,  $O(0)$  間の距離に等しいから  $BC = |b - c|$

$$\begin{aligned} \boxed{4} \quad \frac{1}{x} &= \frac{2}{3 + \sqrt{5}} = \frac{2(3 - \sqrt{5})}{(3 + \sqrt{5})(3 - \sqrt{5})} \\ &= \frac{2(3 - \sqrt{5})}{9 - 5} = \frac{\text{ア}3 - \sqrt{\text{イ}5}}{\text{ウ}2} \end{aligned}$$

$$\text{よって} \quad x + \frac{1}{x} = \frac{3 + \sqrt{5}}{2} + \frac{3 - \sqrt{5}}{2} = \text{エ}3$$

$$\text{また} \quad x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 3^2 - 2 = \text{カ}7$$

$$\begin{aligned} x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right)\left(x^2 - x \cdot \frac{1}{x} + \frac{1}{x^2}\right) \\ &= \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - \text{カ}1\right) \\ &= 3 \cdot (7 - 1) = \text{キ}18 \end{aligned}$$

$\boxed{\text{参考}}$   $x^3 + \frac{1}{x^3}$  の値は、次のようにして求めることもできる。

$$\begin{aligned} x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) \\ &= 3^3 - 3 \cdot 3 = \text{キ}18 \end{aligned}$$

$\boxed{5}$  ① : (b) から (c) への式変形について、 $\sqrt{(a+1)^2} = a+1$  は誤りであり、正しくは

$$\sqrt{(a+1)^2} = |a+1| \text{ である。}$$

よって  $\text{ア} \text{ ①}$